Chy 475—Spring 2009 **Electron spin and angular momentum**
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Electron spin is an intrinsic angular momentum of the electron

By analogy with the commutation relations for "normal" orbital angular momentum which has a natural correspondence with classical angular momentum, we can define a set of angular momentum operators (relabeled with S instead of L) to describe the Pauli principle: the fact that no more than two electrons can inhabit a spatial orbital and these two electrons are distinguishable by their magnetic properties.

Commutators for spin angular momentum

$$\left[\hat{S}^2,\hat{S}_x\right]=0 \qquad \left[\hat{S}^2,\hat{S}_y\right]=0 \qquad \left[\hat{S}^2,\hat{S}_z\right]=0$$
$$\left[\hat{S}_x,\hat{S}_y\right]=i\hbar\hat{S}_z \qquad \left[\hat{S}_y,\hat{S}_z\right]=i\hbar\hat{S}_x \qquad \left[\hat{S}_z,\hat{S}_x\right]=i\hbar\hat{S}_y$$

It turns out that for electrons, the expectation value of $\left\langle \hat{S}_z \right\rangle = \pm\frac{1}{2}\hbar$, implying a spin quantum number $s=\frac{1}{2}$. The expectation value of $\left\langle \hat{S}^2 \right\rangle = \frac{1}{2}\left(\frac{1}{2}+1\right)\hbar^2 = \frac{3}{4}\hbar^2$. So for electrons we have:

$\left\langle \hat{S}^2 \right\rangle = s(s+1)\hbar^2$ with $s=\frac{1}{2}$

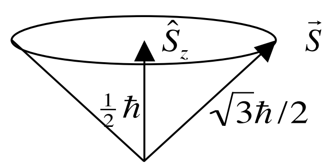
in either of two states α and β where

$\hat{S}_z\alpha = \frac{1}{2}\hbar$ and $\hat{S}_z\beta = -\frac{1}{2}\hbar$.

By analogy with orbital angular momentum, we have

$s=\frac{1}{2}$ and $m_s = \text{-s},\ldots,\text{+s (step by 1)} = -\frac{1}{2},+\frac{1}{2}$

The vector diagram for electron spin looks like:



A. What is the angle between the vector $\vec{S}$ and the z-axis in the α state?

The m_S quantum number provides an additional label for the electron, in addition to *n*, *l*, and $m=m_l$.